Finite-Amplitude Wave Activity and Mean Flow Adjustments in the Stratosphere and Troposphere

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• Synopsis

We utilize finite-amplitude wave activity diagnostic and the Transformed Eulerian Mean (TEM) formalism to quantify large-scale eddies (Rossby waves and synoptic eddies) and associated mean-flow adjustments in the stratosphere and troposphere. Compared to the traditional Eliassen-Palm (E-P) flux divergence, which only gives tendency, the wave activity provides actual magnitudes of eddy and flow adjustments from daily to decadal time scales, with implications for stratosphere-troposphere coupling, baroclinic adjustments, meridional transports, etc.

• Quasigeostrophic (QG) Potential Vorticity (PV) on the Sphere

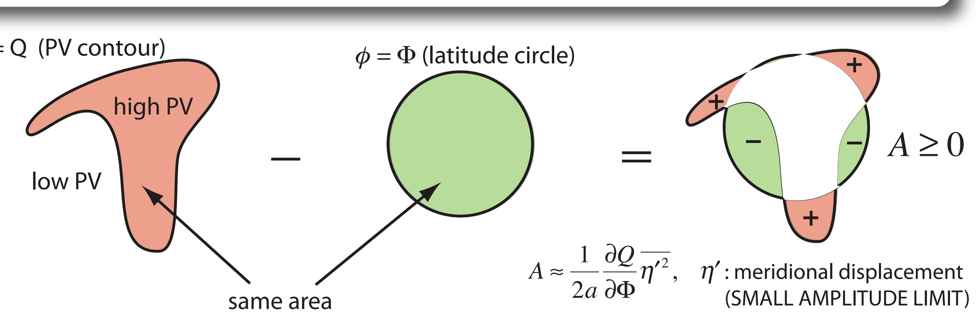
$$q_g \equiv 2\Omega\sin\phi + \frac{1}{a\cos\phi}\left(\frac{\partial}{\partial\lambda}(v\cos\phi) - \frac{\partial}{\partial\phi}(u\cos\phi)\right) + 2\Omega\sin\phi\, e^{z/H}\frac{\partial}{\partial z}\left(\frac{e^{-z/H}(\theta-\tilde{\theta})}{\partial\tilde{\theta}/\partial z}\right)$$

$z = -H\ln(p/1000hPa)$, $\tilde{\theta}(z,t)$: global-mean potential temperature

- QG PV is approximately conserved following geostrophic motion
- We assume that eddies are geostrophically balanced and ignore divergence associated with the latitudinal dependence of the Coriolis parameter

• Finite-Amplitude Wave Activity (interior)

$$\int_{q_g \ge Q} q_g\, dS \quad - \quad \int_{\phi\ge\Phi} q_g\, dS \quad \equiv \quad A(\Phi,t)$$



• Surface Wave Activity

$$\int_{\theta_S \ge \Theta} \theta_S\, dS \quad - \quad \int_{\phi\ge\Phi} \theta_S\, dS \quad \equiv \quad B(\Phi,t)$$

- Surface potential temperature is conserved following geostrophic motion

• Wave Activity and E-P Fux (w/o nonconservative effects)

$$\frac{\partial}{\partial t}(A\cos\phi) = -\cos\phi\,\overline{v'q'_g} = -e^{z/H}\nabla\cdot\mathbf{F} \qquad \frac{\partial}{\partial t}(B\cos\phi) = -\cos\phi\,\overline{v'\theta'_S}$$

$$\mathbf{F} = \left(F^{(\phi)}, F^{(z)}\right) = \left(-e^{-z/H}\cos\phi\,\overline{u'v'},\ e^{-z/H}2\Omega\sin\phi\cos\phi\,\overline{v'\theta'}/(\partial\tilde{\theta}/\partial z)\right)$$: E-P flux

- Combining the above with the QG TEM zonal momentum equation

$$\frac{\partial}{\partial t}(\bar{u}\cos\phi) = 2\Omega\sin\phi\,\bar{v}^*\cos\phi + e^{z/H}\nabla\cdot\mathbf{F}$$

and eliminating the residual mean circulation, one obtains

• PV Gradient Conservation (nonconservative terms on r.h.s.)

$$\frac{\partial}{\partial t}\left(\frac{\partial^2}{\partial\mu^2}\left(\frac{(\bar{u}+A)\cos\phi}{f}\right) + e^{z/H}\frac{\partial}{\partial z}\left(\frac{e^{-z/H}f^2}{N^2(z)}\frac{\partial}{\partial z}\left(\frac{\bar{u}\cos\phi}{f}\right)\right)\right) = n.c. \qquad (1)$$

$f = 2\Omega\sin\phi$, $\mu = \sin\phi$

- Suppose dynamics is fast so that nonconservative terms are negligible. Then one can imagine a hypothetical arrangement of flow in which eddies are removed adiabatcally, i.e., A → 0. At the end of this arrangement, one obtains an eddy-free reference state u_{ref} that satisfies

$$\frac{\partial^2}{\partial\mu^2}\left(\frac{(\bar{u}+A)\cos\phi}{f}\right) + e^{z/H}\frac{\partial}{\partial z}\left(\frac{e^{-z/H}f^2}{N^2(z)}\frac{\partial}{\partial z}\left(\frac{\bar{u}\cos\phi}{f}\right)\right) = \frac{\partial^2}{\partial\mu^2}\left(\frac{u_{ref}\cos\phi}{f}\right) + e^{z/H}\frac{\partial}{\partial z}\left(\frac{e^{-z/H}f^2}{N^2(z)}\frac{\partial}{\partial z}\left(\frac{u_{ref}\cos\phi}{f}\right)\right).$$

Let $\Delta u = \bar{u} - u_{ref}$ and rearrange the terms to obtain

• Adiabatic Mean Flow Adjustment due to Wave Activity

$$\frac{\partial^2}{\partial\mu^2}\left(\frac{\Delta u\cos\phi}{f}\right) + e^{z/H}\frac{\partial}{\partial z}\left(\frac{e^{-z/H}f^2}{N^2(z)}\frac{\partial}{\partial z}\left(\frac{\Delta u\cos\phi}{f}\right)\right) = -\frac{\partial^2}{\partial\mu^2}\left(\frac{A\cos\phi}{f}\right) \qquad (2)$$

with the boundary condition

$$\frac{\partial}{\partial z}\left(\frac{\Delta u\cos\phi}{f}\right) = -\frac{R(1-\mu^2)}{f^2a^2H}e^{-\kappa z_0/H}\frac{\partial^2}{\partial\mu^2}(B\cos\phi) \qquad z_0 = 1km$$

$$\Delta u \quad = \quad -A \quad + \quad \Delta u_R$$

total adjustment = **direct effect of eddy** + **indirect effect of residual circulation**

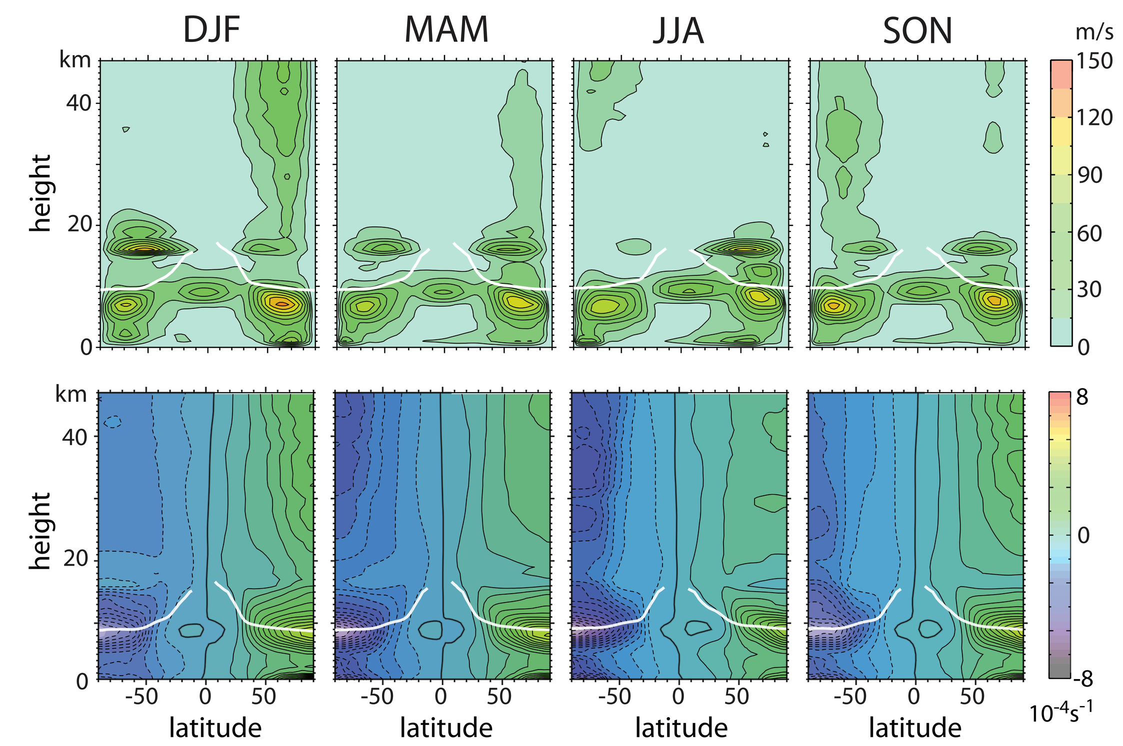
- Alternatively, one can let A → 0 in (1) in a quasi-stationary fashion so $\Delta u = 0$ (this is the paradigm of 'downward control'). Then instead of (2)

$$\frac{\partial^2}{\partial\mu^2}\left(\frac{A\cos\phi}{f}\right) = \int(n.c.)dt \qquad A = \Delta u_R$$

- One can compute the meridional displacement associated with the residual circulation from $\bar{\eta}^* = \Delta u_R / f$ and the vertical displacement from the TEM mass continuity

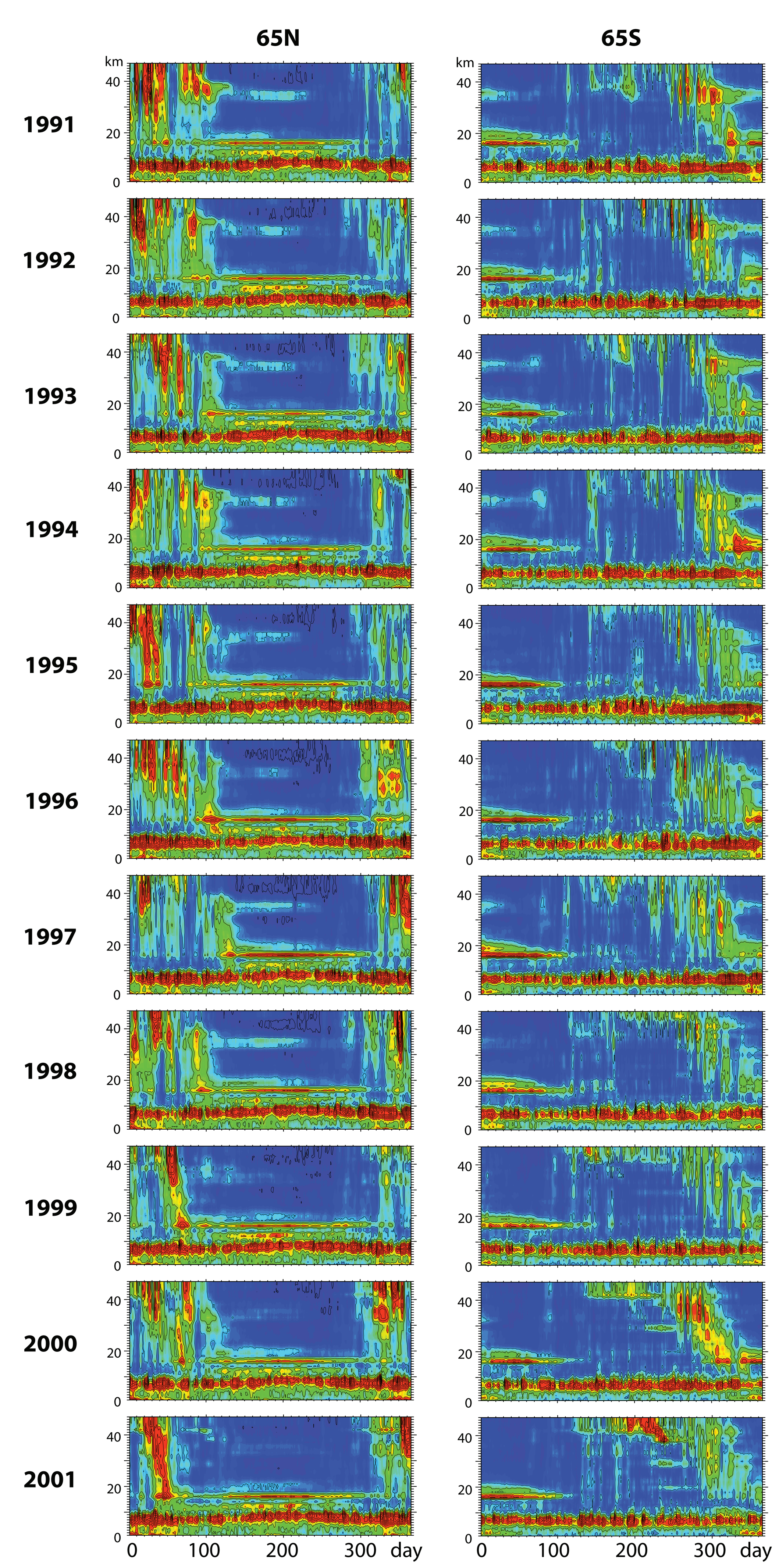
• Seasonal Climatology of A and $\bar{q}_g$

- ECMWF ERA-40 daily data (1979-2001 UTC 1200)



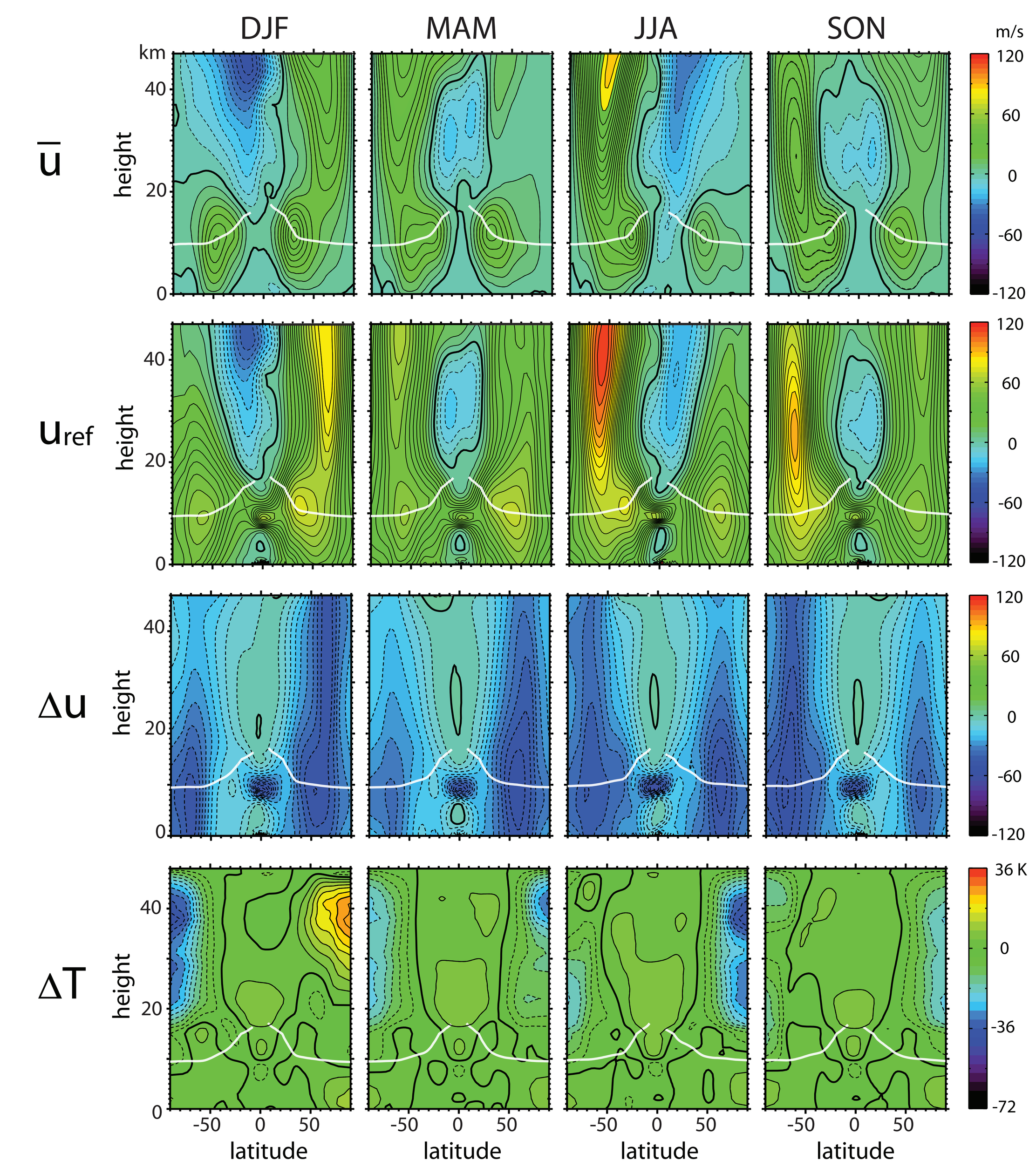
- Wave activity is high in the extratropics, especially just below the tropopause →baroclinic eddies. There is also a weak maximum in the tropical upper troposphere.
- Intrusion of wave activity into the winter/spring stratosphere
- Thin layers of wave activity at the lowermost stratosphere particularly in summer
- Maximum QG PV graients are around extratropical tropopause → temperature contribution
- QG PV gradients are weak (or even reversed) in the tropical upper troposphere, the extratropics of lowermost stratosphere (where the negative meridional temperature gradient nearly cancels positive meridional vorticity gradient), and mid troposphere (critical levels for baroclinic eddies)

• Daily Wave Activity at 65N and 65S (1991-2001, ERA-40)



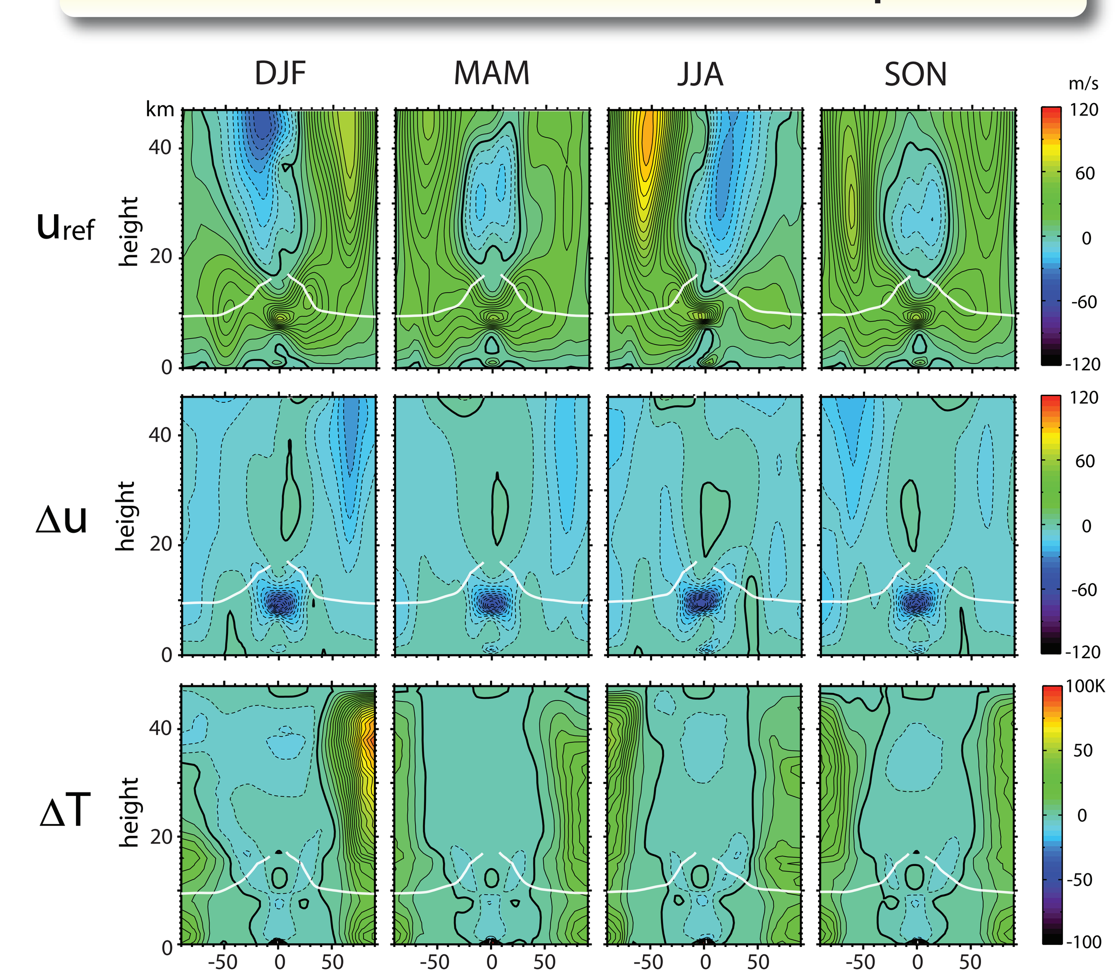
- Strong wave activity in the winter stratosphere tends to occur at high altitudes first, then descend
- Very little wave activity in the summer stratosphere, but the length of inactivity varies from year to year in the NH. In some years the remnant of wave activity lingers into summer at ~ 35 km
- The thin layer of wave activity around 16 km is robust during summer
- Synoptic wave activity just below the tropopause is present throughout the year

• Effects on Mean Flow: Adiabatic Removal (interior + surface)



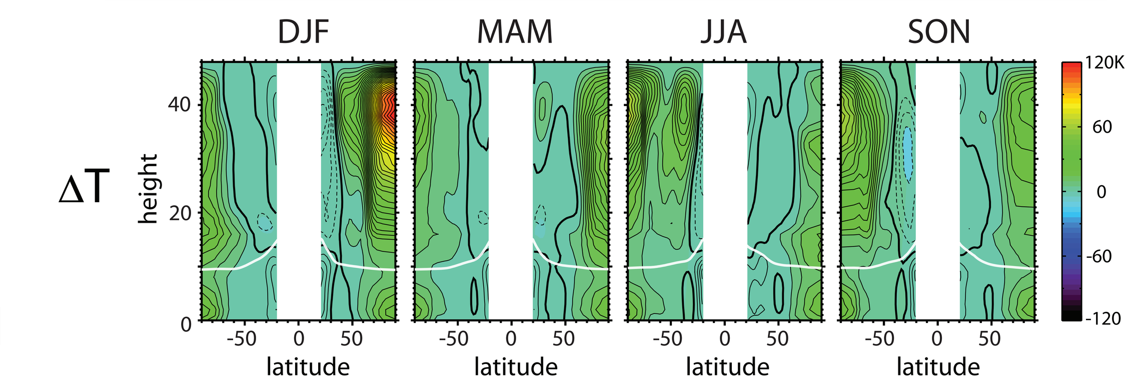
- The net effect of eddy on the mean flow, assuming the eddy arose adiabatically, is a broad deceleration in the extratropics and concentrated deceleration in the tropical upper troposphere. The latter may be exaggerated by the QG assumption
- Maximum deceleration in the extratropics occurs just outside the polar circles and around the tropopause (35 - 50 m/s)
- Extatropical deceleration is nearly barotropic in the troposphere and decays with height in the stratosphere
- After removal of the eddy, surface winds can reach 50 m/s !
- In both hemispheres, eddy warms polar surfaces by up to 10 K
- Except for the NH winter stratosphere, where a 28 K warming occurs, eddy *cools* the high latitudes and warms low latitudes of the stratosphere

• Effects on Mean Flow: Adiabatic Removal / No-Slip Surface



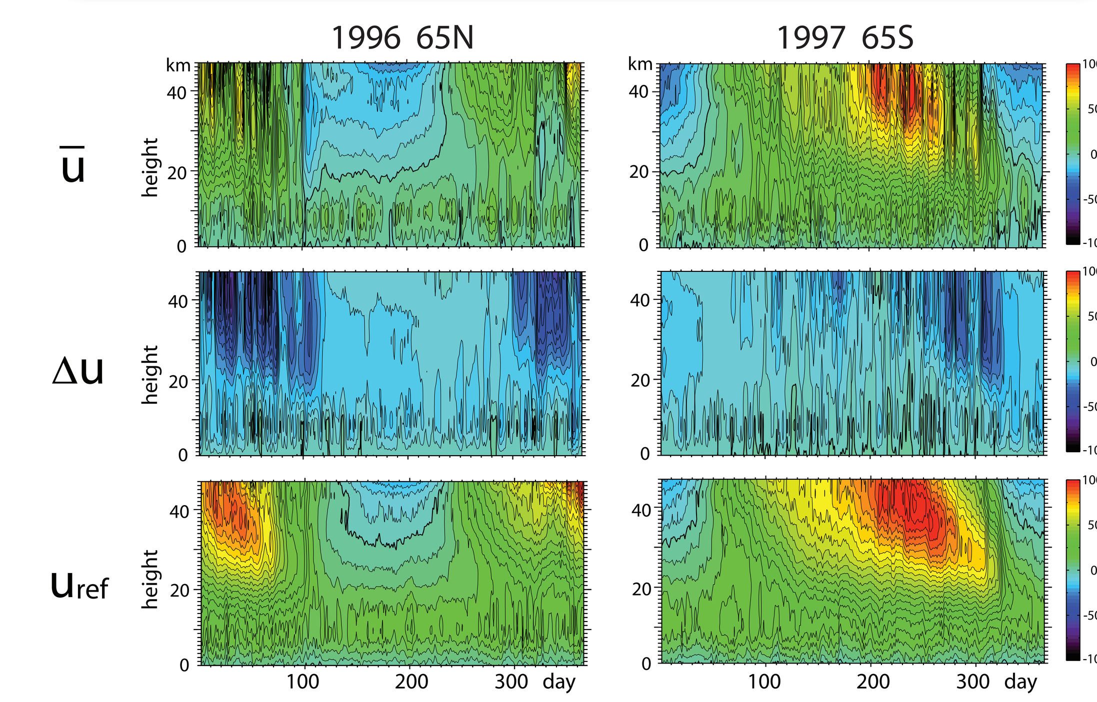
- The assumption here is that eddy turnover is much faster than the diabatic effects in the interior but much slower than the surface friction
- Surface friction kills much of the barotropic deceleration in the adiabatic case
- After removal of the eddy, the tropospheric winds are more vertically sheared than the observed mean flow
- Extratropical deceleration is limited to winter/spring stratosphere
- Now eddy warms the poles and cools the tropics. Polar warming is up to 90 K in the Arctic winter stratosphere and 30 K over the Arctic surface

• Effects on Mean Flow: Quasi-Stationary Removal ($\Delta u = 0$)



- Here we assume that eddy timescale is long, i.e. it is in equilibrium with non-adiabatic effects. Eddy is removed 'slowly' without changing the zonal-mean zonal flow
- The direct and indirect effects of eddy on mean flow cancel, and the indirect effect (residual circulation) is in balance with non-adiabatic forcing
- This is the usual 'downward control' calculation
- Heating of polar regions by eddy is qualitatively similar to the previous case, but low latitudes suffer from noisy results (not shown) due to equatorial singularity

• Construction of (nearly) Eddy-Free Reference State



- The sum of ū and -Δu is the reference state u_{ref}
- Here u_{ref} is computed based on adiabatic eddy removal with no-slip lower boundary condition. u_{ref} removes the advective effect of eddy from the mean flow, making it transparent to large meteorological events like stratospheric sudden warming
- u_{ref} responds largely to non-adiabatic forces and mixing, but it is still constrained to the actual climate state
- Interannual variability and long-term trend in u_{ref} reflect changes in nonconservative process, and thus it is better suited than ū as a climate diagnostic

• Conclusions

- Finite-amplitude wave activity diagnostic may be used in conjunction with the TEM set to quantify the adjustment of flow by large-scale eddy
- The theory is exact for the QG dynamics on the beta plane. On the sphere it is only approximately correct, but we believe that the salient features of our results are valid
- Unlike the E-P flux divergence, the wave activity diagnostic is robust even on a daily snapshot
- Regions of large wave activities: (1) extratropics just below the tropopause, (2) winter and spring stratosphere, (3) thin layer of the extratropical lowermost stratosphere in summer, and (4) tropical upper troposphere
- In the winter stratosphere, large wave activities tend to appear at high altitudes first and then descend. This is always the case with the SH final warming
- Wave activity tends to decelerate the flow on short time scales. However, the pattern of deceleration and residual circulation depends on the lower boundary condition
- Once the eddy effect is removed from the zonal mean flow, the resultant reference state may be used as a slowly varying climate diagnostic
- A large fraction of wave activities around the tropopause are compensated by the indirect effect of residual circulation. This implies a rapid meridional (poleward) transport in those regions
- A more accurate, isentropic version of this analysis is under way

• References

- Andrews and McIntyre (1976 JAS, 1978 JFM)
- Andrews, Holton, and Leovy (1987 Academic Press)
- Haynes (1988 JAS)
- Haynes et al (1991 JAS)
- Magnusdottir and Haynes (1996 JAS)
- McIntyre and Shepherd (1987 JFM)
- Nakamura (1995 JAS)
- Pfeffer (1987 QJRMS)
- Robinson (1986 JAS)
- Solomon and Nakamura (2008 GRL)
- Thuburn and Lagneau (1999 JAS)